NUMERICAL EVALUATION OF THE HEAT TRANSFER AND THE FRICTION DRAG IN A PIPE WITH A TURBULENT FLOW OF A GAS WITH VARIABLE PHYSICAL PROPERTIES

I. S. Olonichev

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The temperature field and the velocity field as well as the local coefficients of heat transfer and friction are determined in the entrance segment of a pipe where a turbulent flow of a gas with variable physical properties stabilizes thermally.

A significant change in the physical properties of a flowing gas can be caused by large temperature gradients at the pipe wall as well as by the occurrence of physicochemical transformations (dissociation, ionization, etc.). Unlike [1-3], this article will deal with the turbulent flow of a compressible gas with variable physical properties in the entrance segment of a circular pipe. The fundamental equations describing a turbulent flow with heat transfer and valid also for the boundary-layer approximations are

$$
\begin{gathered}
\frac{\partial \rho u}{\partial x} + \frac{1}{r} \cdot \frac{\partial \rho v r}{\partial r} = 0; \\
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{1}{r} \cdot \frac{\partial}{\partial r}\left[r(\mu + \varepsilon)\frac{\partial u}{\partial r}\right] - \frac{\partial P}{\partial x}; \\
c_p \rho u \frac{\partial T}{\partial x} + c_p \rho v \frac{\partial T}{\partial r} = \frac{1}{r} \cdot \frac{\partial}{\partial r}\left[r(\lambda + \varkappa)\frac{\partial T}{\partial r}\right] + u\frac{\partial P}{\partial x} + A(\mu + \varepsilon)\left(\frac{\partial u}{\partial r}\right)^2; \\
\frac{\partial P}{\partial r} = 0; \quad \rho,\ \mu,\ \lambda,\ c_p = f(T),
\end{gathered}
\tag{1}
$$

where ε and $\varkappa$ denote the eddy viscosity and the eddy thermal conductivity.

System (1) was solved for the following boundary conditions:

$$
\begin{gathered}
u = u_R, \quad T = T_0, \quad P = P_0 \quad \text{at} \quad x < 0, \quad r_0 \geqslant r \geqslant 0; \\
u = 0, \quad \delta_1 \frac{\partial T}{\partial r} + \delta_2 T = \delta_3, \quad v = 0 \quad \text{at} \quad r = r_0, \quad x \geqslant 0; \\
\frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = 0,
\end{gathered}
\tag{2}
$$

where

$$
u_R = 2.5v^* \ln\left\{[1 + 0.4\eta(1 - R)]\frac{1.5(1 + R)}{1 + 2R^2}\right\}
+ 7.8v^*\left\{1 - \exp\left(-\eta\frac{1 - R}{11}\right) - \frac{\eta}{11}(1 - R)\exp[-0.33\eta_0(1 - R)]\right\}.
$$

As the initial profile served the velocity profile according to Reichardt [4] for an incompressible fluid, this profile being valid for the entire pipe section including the laminar sublayer (unlike the logarithmic or the 1/7th-power profile). The eddy coefficients were calculated from the data in [4] according to

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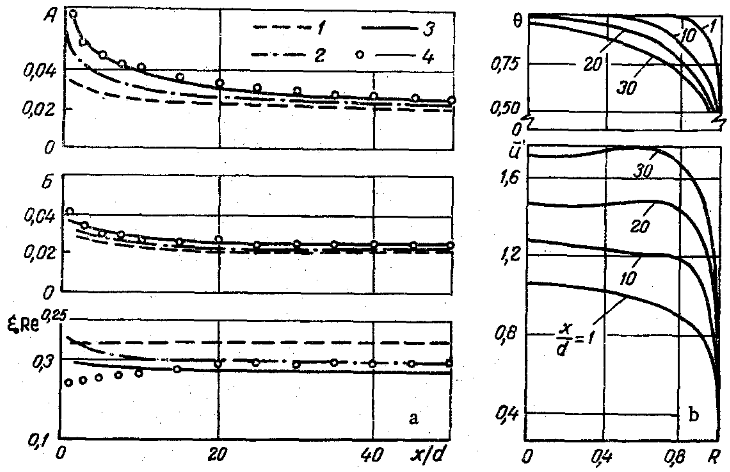

Fig. 1. Parameters $A = Nu_W/Re_W^{0.8}Pr_W^{0.4}$, $B = Nu_G/Re_G^{0.8}$ $Pr^{0.4}\left(\frac{T_W}{T_G}\right)^{-\left\{a+b\lg\frac{T_W}{T_G}\right\}}$, $\xi Re^{0.25}$ and dimensionless profiles of temperature and velocity, as functions of x/d. a: 1) $T_W = 700°K$, $T_0 = 600°K$, $Re = 10^5$; 2) $T_W = 3000°K$, $T_0 = 600°K$, $Re = 10^5$; 3) $T_W = 3300°K$, $T_0 = 300°K$, $Re = 1.3 \cdot 10^5$; 4) $q_W = 1.16 \cdot 10^6$ W/m², $T_0 = 300°K$, $Re = 1.5 \cdot 10^5$; b) $T_W = 3000°K$, $T_0 = 600°K$, $Re = 10^5$.

$$\frac{\varepsilon}{\mu} = 0.4\left(\eta - 11\text{tank}\frac{\eta}{11}\right), \quad 0 \leqslant \eta \leqslant 50,$$

$$\frac{\varepsilon}{\mu} = \frac{0.4}{3}\eta(0.5 + R^2)(1 + R), \quad \eta > 50$$

with the Goldman correction [5] accounting for the variability of physical properties, namely $\bar{\eta} = \int_0^y d\eta$.

The Prandtl number with turbulence was assumed approximately equal to unity.

It has been shown in several studies [1, 2, 3, 10] that these formulas for turbulent transfer during heating of the gas yield numerical results for the stabilization segment of the flow which agree closely with test data obtained with variable physical properties, as during equilibrium dissociation or in the supercritical range of state variables. Here we have assumed that, approximately, these transfer characteristics apply also to the entrance zone of thermal stabilization.

The system of Eqs. (1) with the boundary conditions (2) was solved on a computer by the method of grids. For this, the system was approximated in terms of a two-layer implicit six-point scheme. A uniform scale of steps was used along the x axis and, for better accuracy, a logarithmic scale of steps was used along the radius toward the wall. The resulting systems of algebraic equations were solved by the elimination method combined with iterations.

The scheme of the solution did not differ from that shown in [6] for the case of laminar friction. According to the results of calculations for small temperature differences between wall and gas, the values of the Nusselt number computed on the basis of the temperature fields with the Reynolds number varying from $Re = 10^4$ to 10^6 are in satisfactory agreement (accurately within ±7-9%) with its values computed from the criterial relations on the basis of tests with air [7] and hydrogen [8]. The values of the drag coefficient agree within about 2-4% with those computed according to the Blasius formula.

The numerical values of the Nusselt number Nu and of the drag coefficient for turbulent flow of hydrogen through a circular pipe are shown in Fig. 1.

The computations were made for approximately the same value of the Reynolds number $Re = 10^5$ at the pipe entrance under two different boundary conditions: T_W = const or q_W = const.

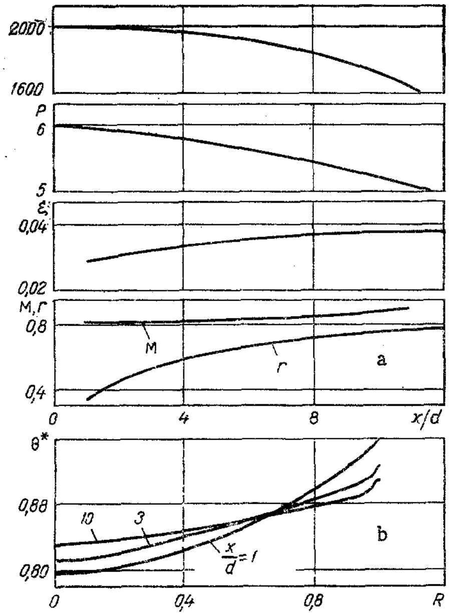

Fig. 2. Static temperature, static pressure, drag coefficient, Mach number at the pipe axis, recovery factor, and profile of the dimensionless stagnation temperature, as functions of x/d at $q_W = 0$: a) $T_0 = 2000°K$, $P = 6$ Pa, $Re = 1.64 \cdot 10^4$; b) $\theta^* = (T^* - T_1)/(T_0 - T_1) = (3000°K - T^*)/1000°K$.

An analysis of these curves indicates that, when q_W = const as well as when T_W = const, the values of $Nu_W/Re_W^{0.8}Pr_W^{0.4}$ spread apart appreciably, depending on the ratio T_W/T_G, with the largest variation within the entrance zone. This finding agrees with the test data in [8, 9], which show that for large values of the temperature ratio ($T_W/T_G \geq 4$) the criterial formula for the Nusselt number must include a factor whose magnitude depends on this ratio.

An introduction of this temperature ratio has made it possible to describe the calculated results accurately enough by a single relation. The basic form of this relation is based on the experimental study in [9] for air with large values of the temperature ratio.

As the thermal flux and the temperature increase, the drag coefficient decreases somewhat below its value for isothermal flow. At $T_W/T_G = 11$ the value of $\xi Re^{0.25}$ differs by 8-10% from the constant coefficient in the Blasius formula.

In Fig. 1 are also shown profiles of the axial velocity component and of the temperature at various sections along a pipe. The variability of physical properties affects most strongly the profiles of axial velocity. Thus, an appreciable density drop caused by a heavy supply of heat will shift the maximum velocity toward the pipe wall.

Some computed characteristics of hydrogen flow through a thermally insulated pipe are shown in Fig. 2. Here the total enthalpy of the stream remains constant, while only the static temperature and pressure vary. It is interesting to note the large variation of the recovery factor r defined as the ratio $r = (T_W - T_G)/(T_{adiab} - T_G)$, which has to do with the initially nonuniform profile of stagnation temperatures.

Calculations have shown that in a sufficiently long pipe r increases approaching closely the value $r = \sqrt[3]{Pr}$ found in the technical literature.

The satisfactory generalization of numerical data (accurate within ±4-6%) by the criterial relations in [9] based on tests indicates that the method described here is applicable to calculations of heat transfer and friction in circular pipes with large values of the temperature ratio ($T_W/T_G \leq 11$).

NOTATION

u, v	are the velocity components;
x, r	are the coordinates;
T_1, T_0	are the wall temperature and the gas temperature at the pipe entrance;
$\eta = v^*y/\nu$;	
r_0, d	are the radius and the diameter of a pipe;
P_0	is the pressure at the pipe entrance;
$R = 2r/d$;	

ρ is the density;

A is the heat equivalent of mechanical work;

M is the Mach number;

Re is the Reynolds number;

Nu is the Nusselt number;

ξ is the drag coefficient;

$\theta = (T-T_1)/(T_0-T_1)$;

Nu_w, Re_w are the Nusselt number and the Reynolds number referred to the wall temperature as the characteristic temperature;

$v^* = \sqrt{\tau_w/\rho}$;

$\eta_0 = v^* r_0/\nu$.

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